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NOTE ON GROUPS OF ORDER* p^2q^2.

By O. E. GLENN, The University of Pennsylvania.

The purpose of this note is to prove the following theorem:

If a group G of order p^2q^2 $(p>q)$ *has five distinct series of composition, all arrangements of the composition factors excepting* (q, p, p, q) *being posssble, then must* $q=2$ *and* $p=3$. *The only existent group is thus of order* 36, *and it is the direct product of the tetrahedron-group and a cyclic group of order* 3.

The invariant subgroup Hq^2p is Abelian, and Hp^2q is a divisible type $\{S_1, S_3\}\{S_2\}$ defined by

$$S_1^p=S_2^p=S_3^q=1, \quad S_1'S_2=S_2S_1, \quad S_3S_2=S_2S_3, \quad S_3'S_1S_3=S_1^a.$$

The Sylow subgroup $I_{q^2}=\{S_3, S_4\}$ is invariant under H_{q^2p} and hence under G. Likewise $\{S_1, S_2\}$ is self-conjugate in G, and these two subgroups contain all the subgroups of G of orders q and p, respectively. The operator S_1 transforms $\{S_3\}$ into p conjugates within $\{S_3, S_4\}$, and since the number of these cannot exceed

$$N_q=\frac{q^2-1}{q-1}=q+1$$

it follows that $p=q+1$, and therefore $p=3$, $q=2$.

Hence $a\equiv -1 \pmod 3$, and the $N_2=3$ subgroups are

$$\{S_3\}, \quad \{S_4\}, \text{ and } \{S_3S_4\}.$$

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*Le Vavavasseur has given in Comptes Rendus, Vol. 128 (1899), p. 152, a list of the groups of order p^2q^2, the proofs having been suppressed. [ED. D.]

If these be transformed by S_2, the number, ρ, remaining invariant, satisfies the congruence $3-\rho\equiv 0 \pmod 3$, and since $S_2^{-1}\{S_3\}S_2=\{S_3\}$, we have $\rho=3$, and $S_2S_4=S_4S_2$. Suppose next that $S_4^{-1}S_1S_4=S_1^xS_2^y$.

$$(S_3S_4)^{-1}S_1(S_3S_4)=S_1^{-x}S_2^{-y}=(S_4S_3)^{-1}S_1(S_4S_3)=S_1^{-x}S_2$$
$$-2y\equiv 0 \pmod 3;\; y\equiv 0 \pmod 3,$$

and as S_4 is not permutable with $S_1 x\equiv -1 \pmod 3$. So that $G=\{S_3, S_4, S_1\}\{S_2\}$ is defined by the relations

$$S_1^3=S_2^3=S_3^2=1,\quad S_1S_2=S_2S_1,\quad S_3S_4=S_4S_3,\quad S_3^{-1}S_1S_3=S_1^{-1},\quad S_2S_3=S_3S_2,$$
$$S_4^{-1}S_1S_4=S_1^{-1},\quad S_2S_4=S_4S_2.$$

The subgroups $\{S_3, S_4, S_1\}$ will be recognized as the abstract form of the tetrahedron rotation group.

TESTS OF DIVISIBILITY BY 7, 13, AND 17.

By MISS ALICE CHURCH, New York City.

1. Seven is an exact divisor of any number of which the units figure doubled differs from the number represented by the remaining digit or digits by zero or by a multiple of seven. Thus, in the number 14, twice 4 are 8; 1 from 8 leaves 7.

In 168, twice 8 are 16; 16 from 16 leaves 0.

In 532, twice 2 are 4; 4 from 53 leaves 49.

Corollary A. When the remainder after subtraction is 0, the original number is a multiple of 3 as well as of 7, therefore is a multiple of 21.

When the number to be treated is so large that the remainder found by subtracting the doubled unit from the rest of the number is too large to be judged by inspection, the same test may be applied to the remainder as to the original number. This process may be repeated until a remainder be found which is small enough to be factored by inspection.

For example, 22134; twice 4 are eight; 8 subtracted from 2213 leaves 2205; twice 5 are 10; 10 from 220 leaves 210; 210 at once appears as a multiple of 7.

Demonstration. Multiplying the units figure by 2 and placing the product in the tens column and ignoring the units figure in the subtraction is really multiplying the units figure by 21, which is a multiple of 7. The test, then, becomes merely the subtraction of a multiple of seven from a possible multiple of 7, or vice versa, and as the difference between multiples must be a multiple, the number tested is divisible by 7 if the difference found is a multiple of 7.